Single Qubit Operations

Definition 11. *The four Pauli gates are the following single-qubit gates:*

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Proposition 5. *The Pauli gates form a basis for $\mathbb{C}^{2\times 2}$, they are Hermitian, and they satisfy the relationship $XYZ = iI$.*

The X, Y, Z gates all correspond to $90°$ rotations, around different axes. The X gate flips a qubit:

$$X|0\rangle = |1\rangle \qquad X|1\rangle = |0\rangle.$$

This is the equivalent of a NOT gate in classical computers. At the same time, the Z gate is also called a phase-flip gate: it leaves $|0\rangle$ unchanged, and maps $|1\rangle$ to $-|1\rangle$.

$$Z|0\rangle = |0\rangle \qquad Z|1\rangle = -|1\rangle.$$

A single-qubit gate that is used in many quantum algorithms is the *Hadamard* gate:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$

The action of H is as follows:

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \qquad H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Superposition

An Introduction to Quantum Computing, Without the Physics, Giacomo Nannicini, 2017 (2020).