Two Qubits

Let us write the basis elements of $(\mathbb{C}^2)^{\otimes 2} = \mathbb{C}^2 \otimes \mathbb{C}^2$*:*

$$|0\rangle_2 = |0\rangle \otimes |0\rangle = |00\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix} \qquad |1\rangle_2 = |0\rangle \otimes |1\rangle = |01\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$$

$$|2\rangle_2 = |1\rangle \otimes |0\rangle = |10\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix} \qquad |3\rangle_2 = |1\rangle \otimes |1\rangle = |11\rangle = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}.$$

$$|x\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$$
$$|y\rangle = \beta_0|0\rangle + \beta_1|1\rangle.$$

that taken as a whole will be in state:

$$|x\rangle \otimes |y\rangle = \alpha_0\beta_0|00\rangle + \alpha_0\beta_1|01\rangle + \alpha_1\beta_0|10\rangle + \alpha_1\beta_1|11\rangle, \quad (1)$$

with the normalization conditions $|\alpha_0|^2 + |\alpha_1|^2 = 1$ and $|\beta_0|^2 + |\beta_1|^2 = 1$. The general state of a 2-qubit register $|\psi\rangle$ is:

$$|\psi\rangle = \gamma_{00}|00\rangle + \gamma_{01}|01\rangle + \gamma_{10}|10\rangle + \gamma_{11}|11\rangle, \quad (2)$$

with normalization condition $|\gamma_{00}|^2 + |\gamma_{01}|^2 + |\gamma_{10}|^2 + |\gamma_{11}|^2 = 1$.

Entanglement

An Introduction to Quantum Computing, Without the Physics, Giacomo Nannicini, 2017 (2020).