A Classical π Machine and Grover's Algorithm

Jiang Liu*

(January 5, 2020)

This paper studies a well-known π machine illustrated by Fig. (1). It is shown that the π machine can compute digits of π if the ratio of block weights, m_2/m_1, satisfies certain conditions, and that dynamics of the π machine is identical to that of Grover's algorithm [1] in quantum computing.

Keywords: Two-Block-Collision π machine; Grover's Algorithm, Quantum Computing

I. INTRODUCTION

Computation of π has never ceased to delight us[1]. One intereting method[6–9] is shown in Fig. (1), where digits of π is computed by counting the number of elastic collisions between two sliding blocks of masses m_1 and m_2, and collisions between block m_1 and a wall. This paper is to take a fresh look at this simple machine.

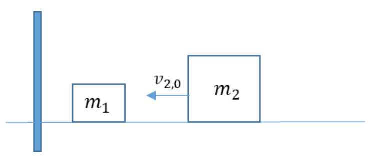

FIG. 1. Initial state of two sliding block elastic collision π machine.

The rest of this paper is organized as follows. Section II shows that calculability of π digits depends on m_2/m_1. Section III shows that dynamics of the π machine is the same as that of Grover's algorithm [1], and that Grover's quantum computing [10] can be visualized classically by the π machine. Our conclusion is summarized in Section V.

II. THE π MACHINE

Consider a weighted velocity space shown by Fig. (2), where

$$\boldsymbol{v}_t \equiv \begin{pmatrix} \sqrt{m_2}v_{2,t} \\ \sqrt{m_1}v_{1,t} \end{pmatrix} \tag{1}$$

and $v_{1,t}$ and $v_{2,t}$ are the velocities of m_1 and m_2 at t. Conservation of kinetic energy requires that $\boldsymbol{v}_t$ be on a circle of radius $|\boldsymbol{v}_t| = \sqrt{m_2}v_{2,0}$. Conservation of momentum implies

$$m_2 v_{2,t+1} + m_1 v_{1,t+1} = m_2 v_{2,t} - m_1 v_{1,t}, \tag{2}$$

where minus sign of $m_1 v_{1,t}$ rises from m_1 bouncing off the wall. In polar coordinates, where $\sqrt{m_2}v_{2,t} = |\boldsymbol{v}_t|\cos\theta_t$

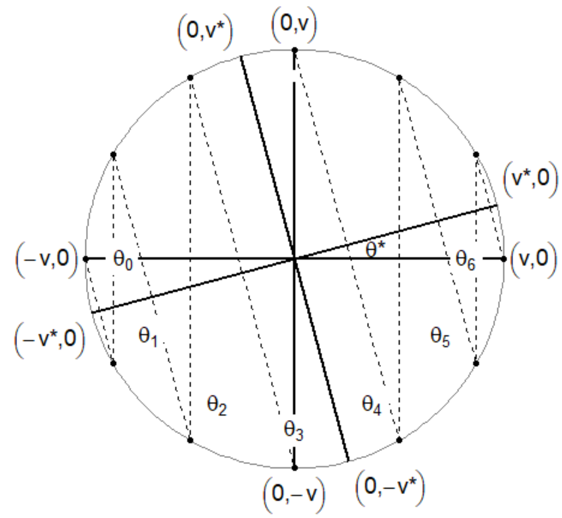

FIG. 2. Geometric representation of a π machine's dynamics whose $\theta^* = \pi/12$. Doted lines parallel to $(0, v) - (0, -v)$ axis, represent m_1 bouncing off the wall. Doted lines parallel to $(0, v^*) - (0, -v^*)$ axis represent $m_1 - m_2$ collision. θ_t is the polar angle of $\boldsymbol{v}_t$ after each $m_1 - m_2$ collision.

and $\sqrt{m_1}v_{1,t} = |\boldsymbol{v}_t|\sin\theta_t$, Eq. (2) can be simplified to

$$\cos(\theta_{t+1} - \theta^*) = \cos(\theta_t + \theta^*), \tag{3}$$

where

$$\sin\theta^* = \sqrt{\frac{m_1}{m_1 + m_2}}. \tag{4}$$

Solution to Eq. (3) with initial condition $\theta_0 = \pi$ is

$$\theta_t = 2t\theta^* + \pi. \tag{5}$$

Collision stops at T when $v_{1,T} \to 0$, i.e., $\theta_T \to 2\pi$. Total number of collisions between $t = 0$ and $t = T$ is $2T$ and from Eq. (5)

$$2T = \left\lfloor \frac{\pi}{\theta^*} \right\rfloor. \tag{6}$$

Clearly, number of collisions prints digits of π if $\theta^* = 10^{-n}$, where n is a positive integer. This happens when $m_2/m_1 = 10^{2n} \gg 1$ (see Eq. (4)). This is the necessary condition for the calculability of the π machine. Otherwise, the π machine cannot produce digits of π.

---

* jiangliu@alumni.cmu.edu

[1] Current world record of π digits is 31.4 trillion by Emma Haruka Iwao using Chudnovsky algorithm [2] on Google Cloud [3–5]

III. THE π MACHINE AND GROVER'S ALGORITHM

That dynamics of the π machine and Grover's algorithm [1] are identical can be viewed most conveniently from Hamiltonian formulation, in which collisions are described by evolution of states of bases $\boldsymbol{v}$ and $\boldsymbol{v^*}$.

In $\boldsymbol{v}$ basis, $\boldsymbol{v}_t$ is an eigenvector of Hamiltonian for m_1 bouncing off the wall. Geometrically, $\boldsymbol{v}_t$ reflects over the $(v,0)$ axis of Fig. (2), i.e.,

$$\boldsymbol{v}_t \rightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \boldsymbol{v}_t. \tag{7}$$

$\boldsymbol{v}$ basis is referred to as computational basis in the literature. It is called computational due to interactions between internal system (blocks of the π machine) and environment(the wall).

$\boldsymbol{v^*}$ basis is related to $\boldsymbol{v}$ basis by a rotation (see Fig. (2)) with θ^* given by Eq. (4)

$$\boldsymbol{v}^*_t = U_{\theta^*}\boldsymbol{v}_t, \tag{8}$$

where

$$U_{\theta^*} = \begin{pmatrix} \cos\theta^* & \sin\theta^* \\ -\sin\theta^* & \cos\theta^* \end{pmatrix}. \tag{9}$$

In $\boldsymbol{v}^*_t$ basis, $\boldsymbol{v}^*_t$ is an eigenvector of Hamiltonian for block collision. Geometrically, $\boldsymbol{v}^*_t$ reflects over the $(v^*,0)$ axis of Fig. (2), i.e.,

$$\boldsymbol{v}^*_{t+1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \boldsymbol{v}^*_t. \tag{10}$$

$\boldsymbol{v^*}$ basis is referred to as canonical basis in the literature. Canonical basis does not involve environment.

It then follows from Eqs. (7) to (10) that dynamics of the π machine in $\boldsymbol{v}$ basis (computational basis) is

$$\boldsymbol{v}_{t+1} = G_{\theta^*}\boldsymbol{v}_t, \tag{11}$$

where

$$G_{\theta^*} = U^\dagger_{\theta^*} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} U_{\theta^*} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{12}$$

Upper component of Eq. (11) is Eq. (3) discussed in the previous section. Lower component of Eq. (11) is the conjugate of Eq. (3), $\sin(\theta_{t+1} - \theta^*) = \sin(\theta_t + \theta^*)$. Both lead to the same result.

Apart from an overall phase of no physical significance, dynamics described by Eqs. (11) and (12) is the same as that of Grover's diffusion operator [1, 10].

As a result, processes described by Fig. (2) for the π machine can be interpolated into Grover's quantum circuits shown by Fig. (3). This interpolation allows us to visualize Grover's quantum computing by the π machine.

Working qubits $|W\rangle$ in Grover's algorithm can be visualized as the wall, searching target $|k\rangle$ as $\sqrt{m_1}v_{1,0}$ and superposition of the rest of qubits as $\sqrt{m_2}v_{2,0}$; Total number of qubits n is determined by $n = \lceil \log_2(1 + m_2/m_1) \rceil$.

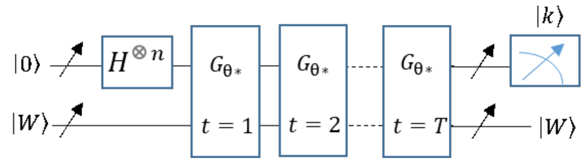

FIG. 3. Quantum π machine simulator (Grover's circuit). $|W\rangle$ is Grover's working qubit representing the wall. $|0\rangle$ is the initial n-qubit state in canonical basis representing block state $(-\boldsymbol{v}, 0)$. $|k\rangle$ is the search target state in computational basis representing block state $\sqrt{m_1}v_{1,0}$.

Grover's diffusion operator G_{θ^*} can be visualized as m_1 bouncing off the wall in computational basis, which plays the same role as Grover's Oracle, i.e., $|k\rangle \rightarrow -|k\rangle$, followed by m_1 and m_2 collision in canonical basis, which plays the same role as phase shift, i.e., $|0\rangle \rightarrow -|0\rangle$. Number of diffusion operators can be visualized as number of collisions.

Most remarkably, basis mixing Eq. (9), which is generated in Grover's algorithm by Hadamard gate rotating qubit along polar axis of Bloch Sphere[11], is a consequence of conservation of kinetic energy and momentum of the π machine.

There is one difference between the π machine and Grover's search. In the π machine, initial state can be prepared and final state can be measured in computational basis alone. In Grover's search, however, initial state $|0\rangle$ is prepared in canonical basis and final state is measured in computational basis. Probability of finding $|k\rangle$ of computational basis, which is equivalent to measuring $\sqrt{m_1}v_{1,t}$, from initial state $|0\rangle$ of canonical basis is

$$Pr(\langle k|0\rangle)_t = \sin^2\theta_t, \tag{13}$$

where $\theta_t = 2t\theta^* + \theta^* \pmod{\pi}$, which has an extra θ^* when compared to Eq. (5).

Apart from the difference of initial state, dynamics of the π machine and Grover's algorithm are identical. Computational complexity of these two processes is the same. In order words, the classical π machine is effectively simulating a type of Grover's quantum computing.

IV. CONCLUSION

Results of this paper can be summarized as follows: digits of π can be computed by the π machine only in limited cases. Dynamics of the π machine is the same as that of Grover's algorithm in all cases. Grover's quantum computing may be classically visualized by the π machine.

After the completion of this work, I noticed from Quanta Magazine [12] a very interesting recent work of Adam Brown [13] on the same subject. Although our approaches are somewhat different, we independently reached the same conclusion.

ACKNOWLEDGMENTS

I would like to thank Ken Li for valuable discussions. He verified numerically the validity of Eqs. (4) and (6).

Any views expressed are mine as an individual and not as a representative speaking for or on behalf of my employer, nor do they represent my employer's positions, strategies or opinions.

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Declarations of interest: None.

[1] L. K. Grover, Proceedings of 28th Annual ACM Symposium on the Theory of Computing , 212 (1996).

[2] D. Chudnovsky and G. Chudnovsky, Approximation and complex multiplication according to ramanujan, Ramanujan revisited: Proceedings of the Centenary Conference (1988).

[3] E. H. Iwao, Most accurate value of π, Guinness World Records Retrieved 2019-03-14 (2019).

[4] Z. Kleinman, Emma Haruka Iwao smashes pi world record with Google help, BBC News, 2019-03-14 (2019).

[5] H. Shaba, A Google employee just shattered the record for Pi calculation. Her name is Emma Haruka Iwao, washingtonpost.com. Washington Post 2019-03-14 (2019).

[6] R. Illner, Hidden Circles and the Digits of π, PI in the SKY (2013).

[7] G. Antonick, The Pi Machine, https://wordplay.blogs.nytimes.com/2014/03/10/pi/ (2014).

[8] X. M. Aretxabaleta, M. Gonchenko, N. Harshman, S. G. Jackson, M. Olshanii, and G. E. Astrakharchik, Two-ball billiard predicts digits of the number PI in non-integer numerical basees, arXiv: 1712.06698v1 [math.DS] (2017).

[9] C. Fraser, https://medium.com/@christopher.fraser/the-pi-machine-the-most-unexpected-answer-to-a-counting-puzzle-b365db558b12, 2019-01-19 (2019).

[10] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, 2010).

[11] F. Bloch, Phys. Rev. **70**, 1946 (1946).

[12] G. Sanderson, How Pi Connects Colliding Blocks to a Quantum Search Algorithm, Quanta Magazine, 21 Jan (2020).

[13] A. R. Brown, Playing Pool with $|\psi\rangle$: from Bouncing Billiards to Quantum Search, arXiv:1912.02207v1 [quant-ph], 4 Dec (2019).