High-level Quantum Computing Emulation for Evaluation of Quantum Algorithms

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High-level quantum computing emulation is described and simulation results are presented for some applications, including phase shift and depolarization errors.

Computations using n qubits can be simulated on a classical computer using an array of 2^n complex values representing the quantum state. This is only feasible for relatively small values of n due to the exponential amount of memory required. And operations which may be done in parallel on a physical quantum computer are simulated using iteration over the state array which requires an exponential amount of time. So a simulation is useful only for evaluating and testing quantum algorithms, not for solving real applications.

In a physical quantum system the 2^n states can not be observed directly. They collapse into a single value when measured, based on the complex amplitude squared, which corresponds to the probability of the measured value. In a simulation however we can manipulate and view the state amplitudes directly, providing insight into the internal operation of quantum algorithms.

Quantum operations can be described mathematically using unitary matrices, but such 2^n-by-2^n matrices are not practical to use directly in a simulation, so alternatives using sparse storage or decision diagram representations may be employed. Here we follow the method described in [1] and use classical function evaluation where appropriate instead of low-level quantum gate simulation, and no matrices are required.

For example, consider the quantum operation of addition mod M: `(a,b)->(a,(a+b)%M)`, where a and b are m-qubit pieces of an n-qubit register q, with n=2m, $N=2^n$, and $M=2^m$. A low-level quantum implementation of addition is described in [2] and is rather complicated. But a high-level simulation can be performed simply using classical addition; in C pseudo-code:

```
t = copy(q); // temporary copy of q register

for( X = 0; X < N; ++X) // for each element of the state array
{
  a = X >> m;       // top m bits
  b = X & mask;     // bottom m bits, mask = 2^m-1 = 0111..1 (m 1's)
  c = (a + b) % M;  // classical addition
  Y = (a << m) | c; // concatenate the bits
  q[Y] = t[X];      // perform the permutation
}
```

This type of reversible computation represents a permutation, e.g. addition with m=1 swaps states 2 and 3:

```
q    a   b  ->  a (a+b)%2
0    0   0      0   0
1    0   1      0   1
2    1   0      1   1
3    1   1      1   0
```

Quantum modular exponentiation and other periodic functions can also be simulated using permutations.

Functions which do not represent a permutation may also be simulated directly at a high-level. The quantum discrete Fourier transform can be simulated by simply performing a classical DFT on the state array coefficients. The phase flip and inversion about the average used in Grover's algorithm can be simulated at a high-level by simply performing the computations; in C pseudo-code:

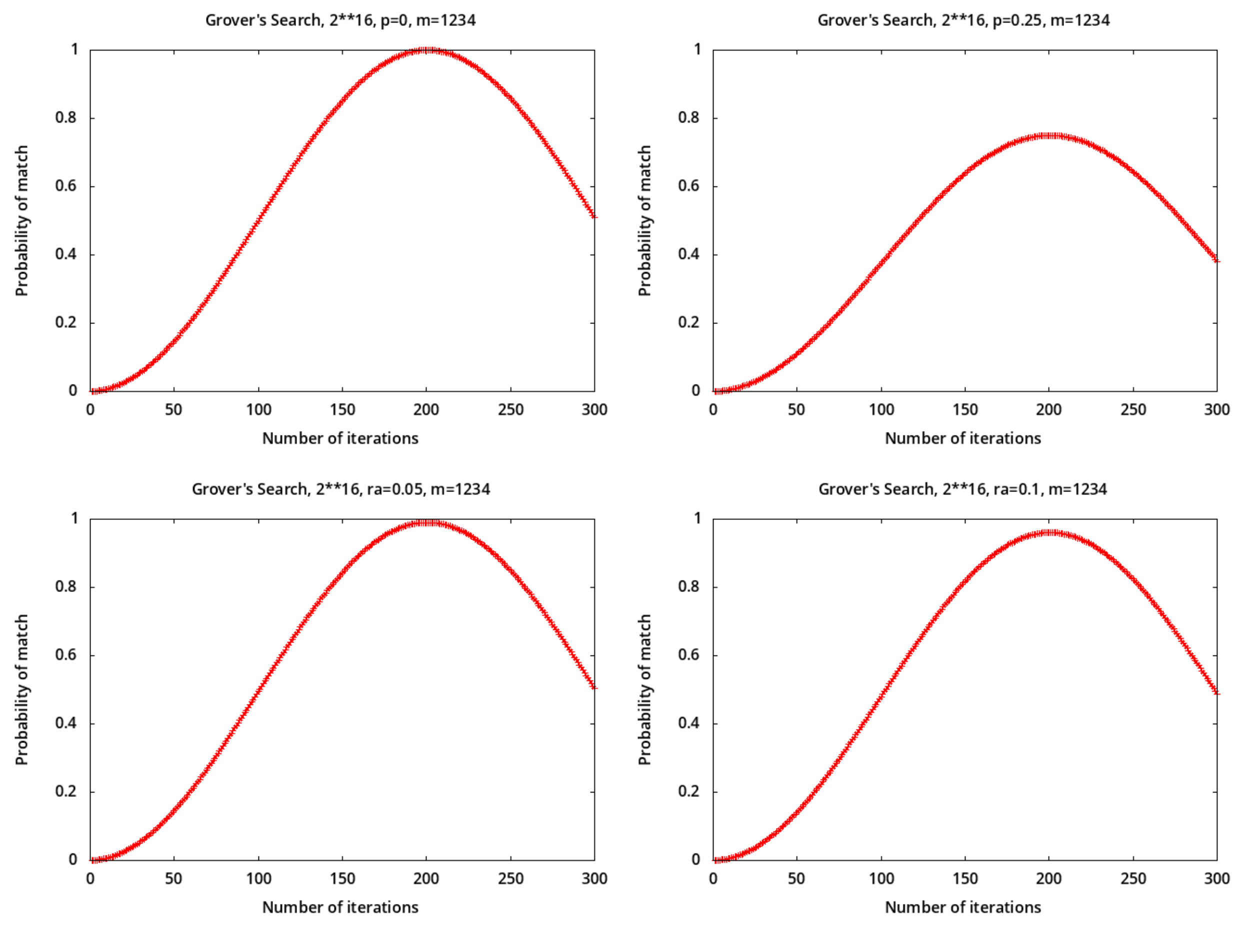
```
    q[m] = -q[m]; // apply f, i.e. flip sign of state m

    a = 0; for( i = 0; i < N; ++i) a += q[i];

    a *= 2.0/N; for( i = 0; i < N; ++i) q[i] = a - q[i]; // inversion about the average
```

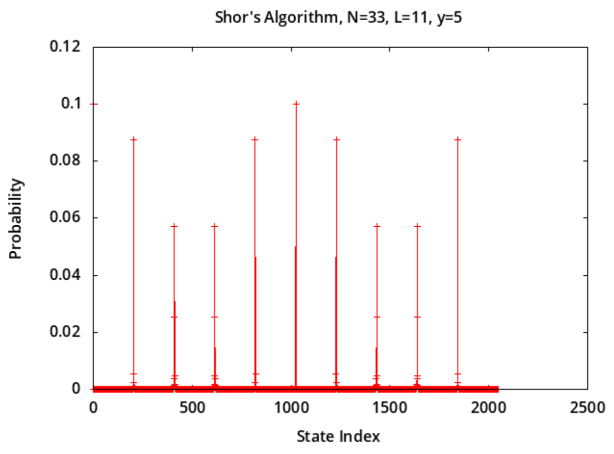
Compared with a low-level quantum circuit for the Grover iteration (from [15]), the high-level simulation does not require any Hadamard transformations or other gates, and also does not require use of auxiliary oracle workspace qubits.

The resulting quantum state is the same regardless of whether it is simulated at a low-level or high-level.

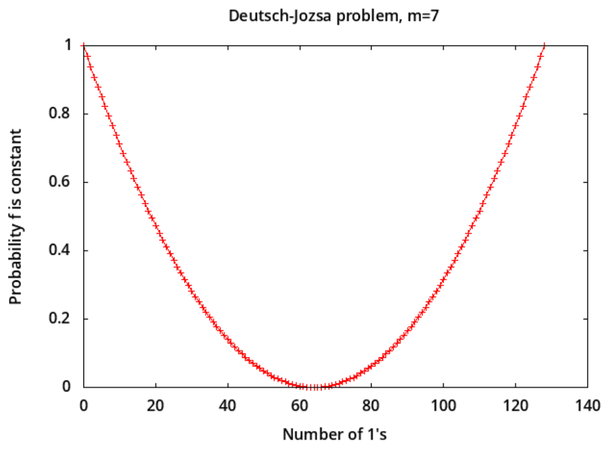


Grover's search - simulation results for searching a 16-bit space, i.e. 2^{16} = 65536 possibilities, which would take $(2^{16})/2$ = 32768 iterations on average using a classical algorithm. The quantum algorithm takes only $(\pi/4)2^{16/2}$ = 201 iterations optimally to find a match with a probability of 0.999988 with no noise (top left plot); additional iterations reduce the probability. In the plot on the top right, depolarizing noise level $p = 0.25$ reduces the probability of match to 0.749995.

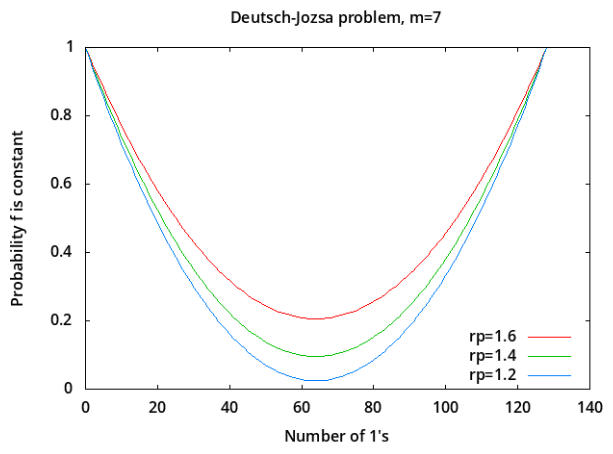
The bottom plots show the effect of errors caused by non-resonant pulses implementing the initial Hadamard transformations (see notes on Magnetic Pulse Error Analysis). For relative error r_a=`0.05` the probability of match after 201 iterations is 0.990014 and for r_a=`0.1` it is 0.960394.

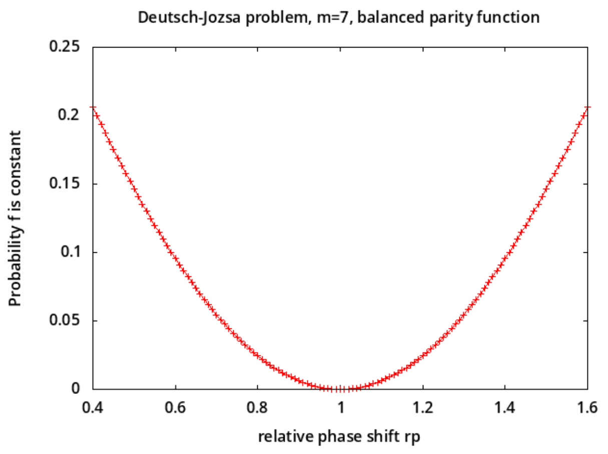


Shor factoring - simulation factoring N=33 using L=11 work qubits and y=5 (order 10). DFT probability peaks occur at (205, 410, 614, 819, ...), and dividing those into 2^{11} produces period estimates (9.9902, 4.9951, 3.3355, 2.5006, ...). Using r = 10 ≈ 9.9902 we have $y^r - 1 = 0$ (mod N), so $(y^{r/2} - 1)*(y^{r/2} + 1) = (5^5 - 1)*(5^5 + 1) = 22*24$; gcd(22,N) = 11, gcd(24,N) = 3, both factors of N.



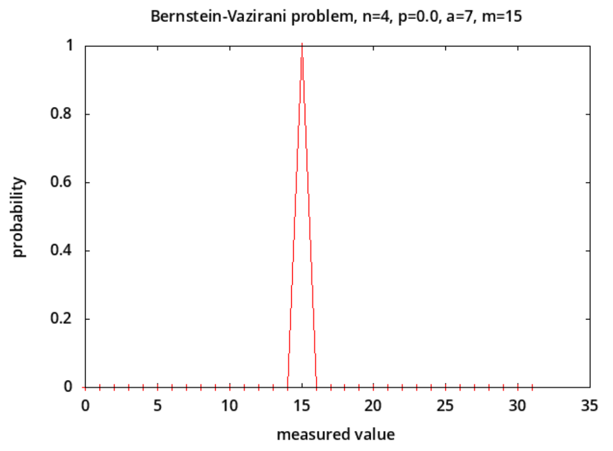
Deutsch-Jozsa problem - simulation results for `P(constant)` vs. number of 1's in the output of a hidden Boolean function. The hidden function is supposed to be either constant (always 0 or always 1) or balanced (0 for half of the possible inputs and 1 for the other half). For a function with m input bits, a classical solution must call the function up to $1+2^{m-1}$ times to determine if it is constant or balanced. But the quantum solution can do it with just one function call, regardless of the size of m. Results shown are for randomly generated functions with 0 to 2^m 1's in their output, so the two end points represent constant functions and only the middle point represents a balanced function. The plot shows the probability varying smoothly between 1 and 0 for the non-constant/non-balanced functions indicating that this algorithm may usefully detect functions which are *almost* constant or balanced.

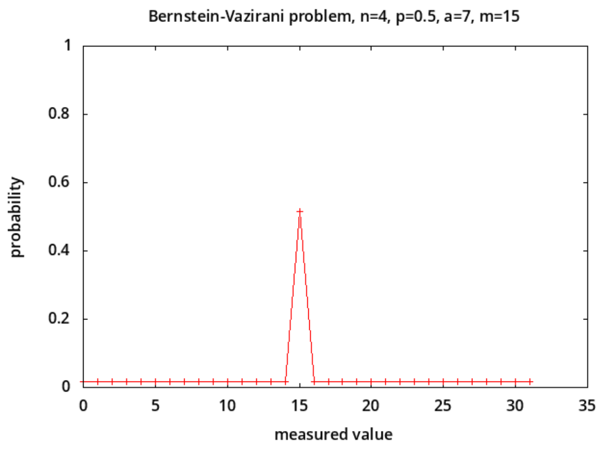


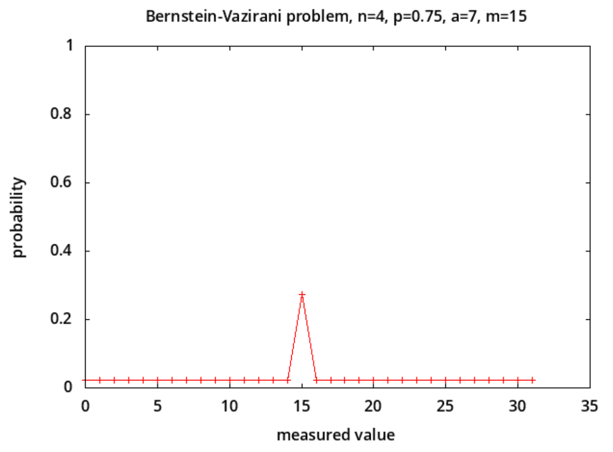


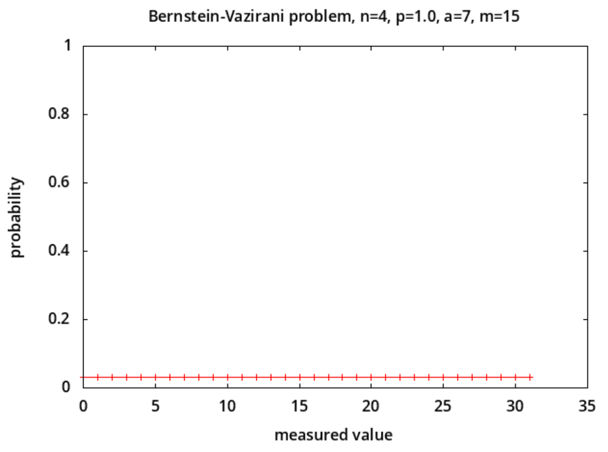
The plots above show `P(constant)` when there is a phase shift error implementing the Hadamard transformations (see notes on Magnetic Pulse Error Analysis). The relative phase shift is r_p`=φ/(π/2)` so r_p`=1` represents no phase shift error. For non-constant functions the phase shift error causes `P(constant)` to increase, so for balanced functions it is greater than 0. In the left plot, for Number of 1's = 64 (i.e. balanced functions), `P(constant)` is `0.0244717, 0.0954915, 0.206107` for r_p `= 1.2, 1.4, 1.6` respectively. The plot on the right shows `P(constant)` for the parity function, which is balanced, for a range of phase shift errors.

Also see notes on Deutsch-Jozsa Coin Analogy.

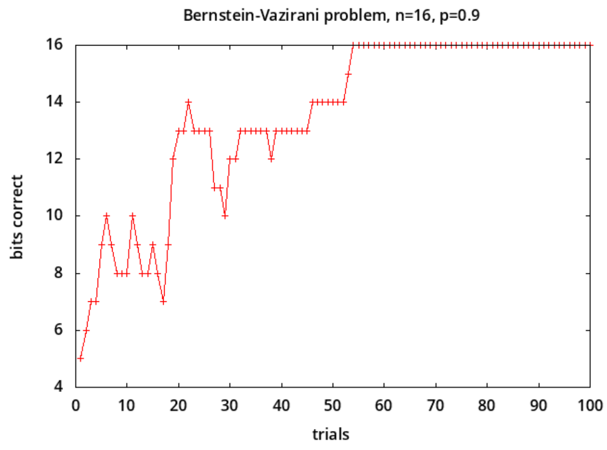


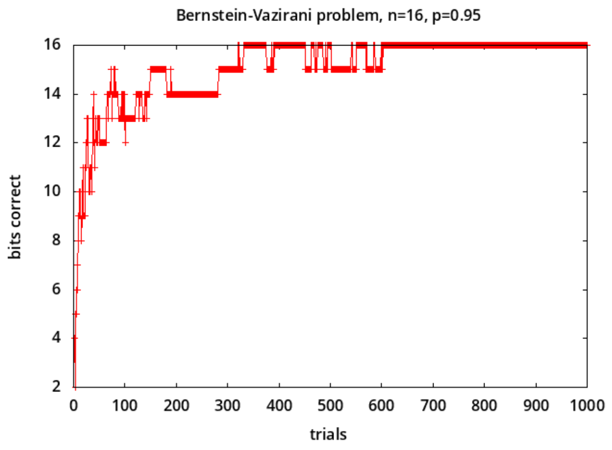






Bernstein-Vazirani problem - determine parameter *a* in the hidden parity function $f(x) = a \cdot x \oplus b$ [7] in the presence of depolarizing noise [12]. This problem is considered to be intractable classically. The probabilities of measured values for noise levels $p = 0.0, 0.5, 0.75, 1.0$ are shown above for $a = 7$ corresponding to correct measured value $m = 2*a + 1 = 15$. With no noise ($p = 0.0$) the correct value is always measured. With 100% noise all possible measured values are equally likely with probability $1/2^{n+1}$ (0.03125 for n=4). The probability of measuring the correct value is linearly related to the noise level: $P(m) = (1-p)*1.0 + p/2^{n+1}$.



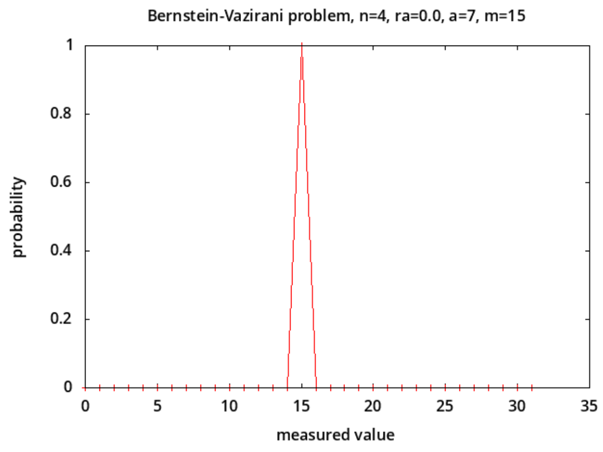


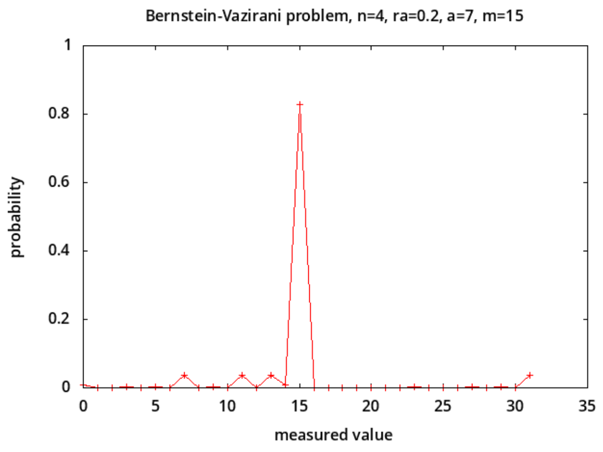
In the simulations shown above, each trial represents a measurement, and the 16-bit parameter is estimated using a bit-wise majority vote following [12]. In the examples shown, for noise level 0.9 all bits are correct after about 60 trials, and for noise level 0.95 all bits are correct after about 650 trials.

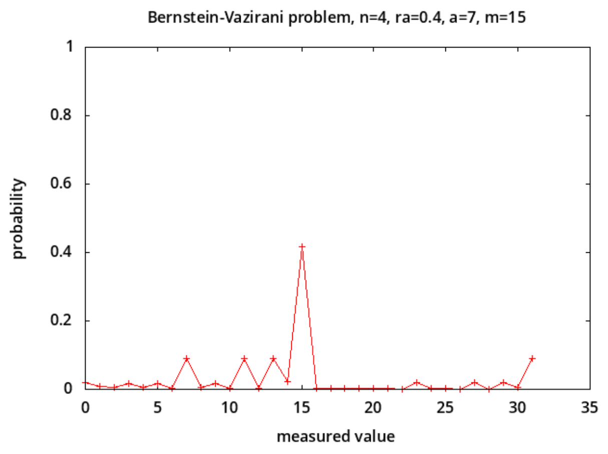
The Bernstein-Vazirani algorithm is performed just once, and then repeated measurements are simulated based on the state probabilities (amplitudes squared). Unlike a physical quantum system, simulated measurements do not collapse the state, so multiple measurements may be performed without redoing the algorithm.

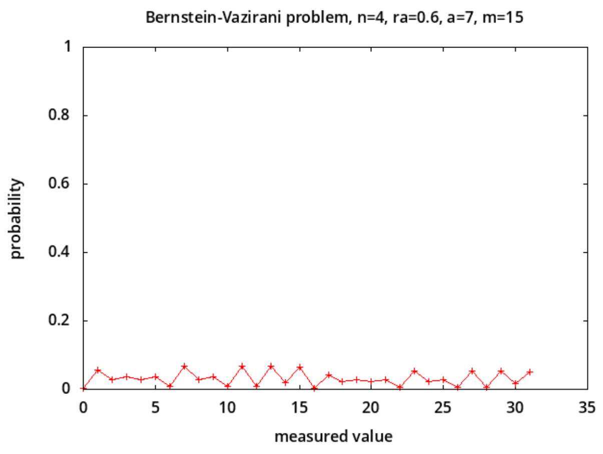
Depolarization produces a mixed state which can not be represented using a state vector (see notes on Depolarization and Mixed States). And according to [12], depolarization should be performed before the final Hadamard transformation. But unitary transformations have no effect on a depolarized density matrix, since $\mathbf{U*I*U^\dagger} = \mathbf{U*U^\dagger} = \mathbf{I}$.

So depolarization is simulated after **H**, just before measuring, by creating a state which will lead to proper measurements, changing each state amplitude A using a convex combination, so $|A|$ becomes $sqrt((1-p)*|A|^2 + p*h^2)$ and the phase becomes $(1-p)*\angle A + p*0$, where $h^2 = 1/2^{n+1}$.









The plots above show the probabilities of measured values with non-resonant pulses implementing the Hadamard transformations for different relative error levels r_a. (see notes on Magnetic Pulse Error Analysis). With no error ($r_a = 0.0$) the correct value is always measured. For $r_a = 0.2, 0.4, 0.6$ the probability of measuring the correct value is 0.828412, 0.417071, 0.0647358 respectively.

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