

It can be many way to transfer 3d world to 2d plane . I think the basic one is planar projection what I showed in the picture. I made steps for beginners and to avoided high mathematics for clear understanding. I believe rotating is next step after you understand all the points to transfer one point from 3D into 2D .

I would like to offer which mathematics in this conversion. As you can see in the figure you want to find m and n values for screen as integer.

1- Define left top point of screen in the plane. $S_1(x_1, y_1, z_1)$

2- Define right top point of screen in the plane. $S_2(x_2, y_2, z_2)$. , The width of screen must satisfy $W = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$. you can select $z_1 = z_2$ for straight view thus W can be $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

3- Define left botton point of screen in the plane. $S_3(x_3, y_3, z_3)$. The height of screen must satisfy $H = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2 + (z_3 - z_1)^2}$ and also we know that screen rectangle. it must satisfy $S_2S_1.S_3S_1 = 0 \longrightarrow (x_2 - x_1)(x_3 - x_1) + (y_2 - y_1)(y_3 - y_1) + (z_2 - z_1)(z_3 - z_1) = 0$
Note: If we want straight view, we can select that $x_1 = x_3$ and $y_1 = y_3$ thus H will be $z_1 - z_3$

4- Find the middle point of screen $M(x_0, y_0, z_0) = (\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}, \frac{z_2+z_3}{2})$

5- Define how far camera will be from screen. (h)

6-Find camera point: $C(x_c, y_c, z_c)$ you need to find plane equation : $ax + by + cz = 1$ three point is enough to define a plane. Thus

-Put Point S_1 : $ax_1 + by_1 + cz_1 = 1$

-Put Point S_2 : $ax_2 + by_2 + cz_3 = 1$

-Put Point M : $ax_0 + by_0 + cz_0 = 1$

Solve a, b, c and find normalization vector that right angle to the plane $N = (a_n, b_n, c_n) = (\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}})$

$C(x_c, y_c, z_c) = (x_0 + ha_n, y_0 + hb_n, z_0 + hc_n)$

7-Find A' that projection of point A on the screen plane.

-Define line between point $C(x_c, y_c, z_c)$ and point $A(x_a, y_a, z_a)$:

$$\frac{x-x_a}{x_c-x_a} = \frac{y-y_a}{y_c-y_a} = \frac{z-z_a}{z_c-z_a} = k$$

and put x, y, z into the plane equation $(ax + by + cz = 1)$ and get an equation depends on k

and then solve k . You can get $A'(x'_a, y'_a, z'_a)$ from $\frac{x-x_a}{x_c-x_a} = \frac{y-y_a}{y_c-y_a} = \frac{z-z_a}{z_c-z_a} = k$ after solving k .

8-Find m, n : $\cos u = \frac{S_2 S_1 \cdot A' S_1}{|S_2 S_1| |A' S_1|} = \frac{(x_2-x_1)(x'_a-x_1)+(y_2-y_1)(y'_a-y_1)+(z_2-z_1)(z'_a-z_1)}{W \sqrt{(x'_a-x_1)^2+(y'_a-y_1)^2+(z'_a-z_1)^2}}$. If $\cos u < 0$ then A' is out of screen. we cannot draw in out 2D screen. If $\cos u > 0$ then $m = \frac{\sqrt{(x'_a-x_1)^2+(y'_a-y_1)^2+(z'_a-z_1)^2} \cos u}{W} = \frac{S_2 S_1 \cdot A' S_1}{W}$

$$n = \sqrt{(x'_a-x_1)^2+(y'_a-y_1)^2+(z'_a-z_1)^2} \sin u$$

We need integers if so the must ignore after point for m and n to get integer values.

if $m > W$ and $n > H$ then we cannot draw the point in screen.

Example:

1: $S_1(400, 400, 400)$

2: if our screen width: 800 pixel $S_2(880, 1040, 400)$ $z_1 = z_2$ for straight view thus $W = \sqrt{(880-400)^2+(1040-400)^2} = 800$

3: $S_3(400, 400, -200)$ thus $H = 600$

4: $M(x_0, y_0, z_0) = (640, 720, 100)$

5: Define how far camera will be from screen. I selected $h = 50$. if h is smaller more area can be seen in screen. It can be changed in software as parameter to get the best view for the screen.

6: Find camera point: $C(x_c, y_c, z_c)$ you need to find plane equation : $ax + by + cz = 1$

$$400a + 400b + 400c = 1$$

$$880a + 1040b + 400c = 1$$

$$640a + 720b + 100c = 1$$

here solution that wolfram helped:

$$a = \frac{1}{100}$$

$$b = -\frac{3}{400}$$

$$c = 0$$

Thus the plane equation of the screen is $\frac{1}{100}x - \frac{3}{400}y = 1$

$$4x - 3y = 400$$

$$N = (a_n, b_n, c_n) = (\frac{4}{5}, -\frac{3}{5}, 0)$$

$$C(x_c, y_c, z_c) = (640 + 50 \cdot \frac{4}{5}, 720 - 50 \cdot \frac{3}{5}, 100) = (680, 690, 100)$$

7-Find A' that projection of point A on the screen plane. A given $(0, 400, 400)$

$$\frac{x}{680} = \frac{y-400}{690-400} = \frac{z-400}{100-400} = k$$

$$4x - 3y = 400$$

$$4(680k) - 3(290k + 400) = 400$$

$$k = \frac{32}{37}$$

$$x = 680k = 680 \frac{32}{37} = \frac{21760}{37} \approx 588,10$$

$$y = 590k + 400 = 590 \frac{32}{37} + 400 = \frac{33680}{37} \approx 910,27$$

$$z = -300k + 400 = -300 \frac{32}{37} + 400 = \frac{5200}{37} \approx 140,54$$

$$8- \cos u = \frac{S_2 S_1 \cdot A' S_1}{|S_2 S_1| |A' S_1|} = \frac{480.188,10 + 640.510,27}{800.602,558} \approx 0,8647$$

$$\sin u \approx 0,5022$$

$$0,8647.602,558 = 521.0319026 \longrightarrow m = 521$$

$$0,5022.602,558 = 302.6046276 \longrightarrow n = 303$$

m and n are selected integer because we needed to find pixel values of the screen.

The example is to demonstrate only one point transfer from 3D to 2D. I hope It will give you a start point to use 3d analytic geometry tools for your purpose.