Multiple User Maximum Likelihood Based Sequence Estimation for Unknown, Frequency Selective, Fast Time Varying Channels *

Diane Pytel Kevin Buckley Richard Perry

Department of Electrical and Computer Engineering
Villanova University
Villanova, PA 19085
E-mail: buckley(perry)@ece.villanova.edu

Abstract

In this paper maximum likelihood sequence estimation (MLSE) is applied to unknown, frequency selective, fast time-varying channels. Assuming a first order Gauss-Markov probabilistic model for the FIR intersymbol interference (ISI) channels, a Maximum A-Posterior (MAP) or ML cost function for the communication symbols is obtained by integration over the channel FIR parameters. A reduced-complexity, time-recursive sequence estimation algorithm is then describeded which is based on Per Survivor Precessing (PSP) and Reduced State Sequence Estimation (RSSE).

1. Introduction

In this paper we consider the K user direct sequence CDMA communications problem where a single-antenna receives the superposition of the users' signals through fast time-varying ISI channels and additive white Gaussian noise. Whereas MLSE is widely used in digital communication systems to estimate a transmitted data sequence for the single user, known channel case, it is not presently considered practical for the multiuser case considered here. For the known multiuser ISI channel case, the MLSE receiver is well known to be a straightforward extension of the MLSE algorithm described by Verdu [1] for the asynchronous (no ISI) case. Though the Verdu algorithm employs the computationally efficient Viterbi algorithm, the number of trellis states (2^{K-1} for binary symbols) has been considered prohibitively large for many multiuser applications. The presence of unknown, fast time-varying, ISI channels represents a significant complication.

Several recent developments in digital communications have potential utility for a MLSE based solution to the problem addressed here. *First*, for single user applications, several exhaustive search algorithms have been proposed for unknown, time-varying, ISI channels. These include joint channel/sequence estimators where for each possible symbol sequence the channel is estimated using a LMS or RLS algorithm (Seshadri [2], Kobo and Fujino [3]) or a Kalman filter (Gustafsson and Wahlberg [4]). Alternatively, Chen, et al. [5] developed an optimum sequence estimator by assigning a time-varying stochastic model for the channel and marginalizing over the channel model parameters to obtain a ML cost for the symbol sequence.

Although these exhaustive search algorithms are computationally unrealistic for long symbol sequences, a *second* development – Per Survivor Processing (PSP) (Raheli, et al. [6]) – provides an effective, suboptimal means of *pruning* the number of sequences for which channel estimates and/or sequence costs have to be computed. Efficient, effective, suboptimum PSP algorithms were suggested in [2, 3, 4] and [5]. These PSP algorithms are formulated using a trellis representation of hypothesized symbol sequences, and a list Viterbi approach to pruning, where only the L best paths into each trellis state are kept for further consideration. For multiuser CDMA, Fonollosa, et al. [7] formulated the MLSE problem, and proposed a PSP-like algorithm which employs LMS based time-varying channel estimation. However, even with PSP, the memory and computational requirements are prohibitive for even moderate numbers of users. In terms of a trellis formulation of the problem, the number of trellis states, and therefore trellis branches, grows exponentially with the number of users K and the ISI channel memory depth $M-1$.

A *third* development which addresses this exponential growth of memory and computation is Reduced State Sequence Estimation (RSSE). Duel-Hallen and Heegard [8] proposed a decision-feedback RSSE approach in which tentative past symbol decisions are used to reduce the number of unknown symbols represented by the trellis states (i.e. by

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reducing the number of assumed unknown symbols in the channel memory). Eyuboglu and Qureshi [9] also proposed trellis state reduction, coined the term RSSE, and considered trellis state reduction approaches based in symbol clustering (applicable for modulation schemes with many symbols) and decision-feedback. More recently, Skelton and Taylor [10] developed a multiuser CDMA reduced-complexity approach based on trellis substages and state clustering RSSE.

Finally, with digital communications hardware advancements following Moore's law, practical multiuser MLSE is coming closer to reality.

In this paper we describe the unknown, fast time-varying, frequency-selective multiuser CDMA problem. We will develop the MLSE solution by marginalizing over the parameters of a Gauss-Markov time-varying model of the multiuser channels. We then investigate PSP and RSSE based reduced complexity algorithms.

2. Multiuser System Model

Here we employ the multiuser, chip-rate sampled, CDMA ISI channel model described in detail in Wang and Poor [11], extended to time-varying channels. Index the users as $k = 1, 2, \cdots, K$ and chips per symbol as $j = 1, 2, \cdots, N$. The users' signaling waveforms upon which the user's bits are impressed can be written as

$$s_k(t) = \sum_{j=1}^{N} c_k(j)\Psi(t - jT_c) \tag{1}$$

where N is the processing gain, $c_k(j)$ is the pseudorandom code assigned to each user and Ψ is the normalized chip waveform of duration $T_c = T/N$. At the receiver, assume chip-rate sampling over symbol times $i = 1, 2, \cdots, n$. The observed data is:

$$\begin{aligned} r_i(j) \;&=\; \sum_{k=1}^{K} \mathbf{a}_i^T(k)\mathbf{h}_i(j,k) \;+\; n_i(j) \;; \\ &\qquad i = 1, 2, \cdots, n \;;\;\; j = 1, 2, \cdots, N, \end{aligned} \tag{2}$$

where the $n_i(j)$ are zero-mean Gaussian noise samples uncorrelated across both i and j with variance σ^2, $\mathbf{h}_i(j,k)$ is the $M \times 1$ dimensional coefficient vector for the time-varying FIR channel for the k^{th} user and j^{th} chip position at symbol time i, $M-1$ is the memory depth (in symbols) of the channels, and $\mathbf{a}_i(k)$ is the vector of the k^{th} user symbols in the FIR channel at symbol time i. The $\mathbf{h}_i(j,k)$ are unknown (and time-varying at the symbol rate) and the problem is to optimally estimate the K users' symbols up to time n. Defining $\mathbf{a}_i \;= [\mathbf{a}_i^T(1), \mathbf{a}_i^T(2), \cdots, \mathbf{a}_i^T(K)]^T$ and $\mathbf{h}_i(j) \;= [\mathbf{h}_i^T(j,1), \mathbf{h}_i^T(j,2), \cdots, \mathbf{h}_i^T(j,K)]^T$, we have

$$r_i(j) \;=\; \mathbf{a}_i^T\mathbf{h}_i(j) \;+\; n_i(j) \;\; . \tag{3}$$

The observation over the symbol interval i is

$$\mathbf{r}_i \;=\; [r_i(1),\; r_i(2), \cdots, r_i(N)]^T \;\; , \tag{4}$$

and the observation up to symbol time n is

$$\mathbf{r}^n \;=\; [\mathbf{r}_1^T, \mathbf{r}_2^T, \cdots, \mathbf{r}_n^T]^T \; . \tag{5}$$

Let $\mathbf{A}^n$ and $\mathbf{H}^n$ denote the matrices of, respectively, all users' symbols and all time-varying FIR channel coefficients up to time n. (The constructions of $\mathbf{A}^n$ and $\mathbf{H}^n$ are not important in this discussion.) The joint probability density function of $\mathbf{r}^n$, conditioned on $\mathbf{A}^n$ and $\mathbf{H}^n$, is

$$f(\mathbf{r}^n|\mathbf{H}^n, \mathbf{A}^n) \;=\; \frac{1}{(\pi\sigma^2)^{nN}} \prod_{i=1}^{n}\prod_{j=1}^{N} e^{-|r_i(j)-\mathbf{a}_i^T\mathbf{h}_i(j)|^2/\sigma^2} . \tag{6}$$

Define the $MKN \times 1$ dimensional vector of all channel coefficients at symbol time i as

$$\mathbf{h}_i \;=\; [\mathbf{h}_i^T(1), \mathbf{h}_i^T(2), \cdots, \mathbf{h}_i^T(N)]^T \tag{7}$$

and consider the $N \times MKN$ dimensional symbol matrix at symbol time i

$$\mathbf{A}_i \;=\; \begin{bmatrix} \mathbf{a}_i^T & \mathbf{0}_{1\times MK} & \mathbf{0}_{1\times MK} & \cdot & \cdot & \mathbf{0}_{1\times MK} \\ \mathbf{0}_{1\times MK} & \mathbf{a}_i^T & \mathbf{0}_{1\times MK} & \cdot & \cdot & \mathbf{0}_{1\times MK} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \mathbf{0}_{1\times MK} & \mathbf{0}_{1\times MK} & \mathbf{0}_{1\times MK} & \cdot & \cdot & \mathbf{a}_i^T \end{bmatrix} \tag{8}$$

where $\mathbf{0}_{1\times MK}$ is the $1 \times MK$ row vector of zeros. With these we have that

$$f(\mathbf{r}^n|\mathbf{H}^n, \mathbf{A}^n) \;=\; \frac{1}{(\pi\sigma^2)^{nN}} \prod_{i=1}^{n} e^{-||\mathbf{r}_i-\mathbf{A}_i\mathbf{h}_i||^2/\sigma^2} \;\; . \tag{9}$$

For the ML estimate of $\mathbf{A}^n$ we determine the $\mathbf{A}^n$ which maximizes

$$f(\mathbf{r}^n|\mathbf{A}^n) \;=\; \int_{\mathbf{H}^n} f(\mathbf{r}^n|\mathbf{H}^n, \mathbf{A}^n)\; f(\mathbf{H}^n)\; d\mathbf{H}^n \;\; . \tag{10}$$

3. MLSE with Gauss-Markov Channels

For a fast time-varying channel with Gauss-Markov fading parameters, let α represent a complex first-order Markov factor such that $|\alpha| = e^{-\omega T}$ where T is the symbol period and $\frac{\omega}{\pi}$ is the Doppler spread. Let the time-varying channel coefficients follow a Gauss-Markov distribution such that, at time i, $\mathbf{h}_i = \alpha\mathbf{h}_{i-1} + \mathbf{v}_i$, where $\mathbf{v}_i$ is complex, white, and Gaussian with zero mean and covariance $\mathbf{C}$. In this case, $f(\mathbf{h}_i|\mathbf{h}_{i-1})$ is Gaussian with mean $\alpha\mathbf{h}_{i-1}$ and covariance $\mathbf{C}$. $f(\mathbf{H}^n)$ can be written as

$$f(\mathbf{H}^n) = \prod_{i=1}^{n} f(\mathbf{h}_i|\mathbf{h}_{i-1}) \; . \tag{11}$$

We assume that the initial channel coefficients are known, and they are represented as $\mathbf{h}_0$. Then

$$f(\mathbf{r}^n|\mathbf{A}^n) \doteq \prod_{i=1}^{n}\int_{\mathbf{h}_i} e^{-Q_i} d\mathbf{h}_i \ , \tag{12}$$

$Q_i = ||\mathbf{r}_i - \mathbf{A}_i\mathbf{h}_i||^2/\sigma^2 - (\mathbf{h}_i - \alpha\mathbf{h}_{i-1})^H\mathbf{C}^{-1}(\mathbf{h}_i - \alpha\mathbf{h}_{i-1})$. We can marginalize over the channel coefficient distribution step by step. The integral over $\mathbf{h}_1$ is

$$I_1 = \int_{\mathbf{h}_1} e^{-E_1} d\mathbf{h}_1 \ , \tag{13}$$

with

$$\begin{aligned} E_1 \quad &= \quad (\mathbf{h}_1 - \alpha\mathbf{h}_0)^H\mathbf{C}^{-1}(\mathbf{h}_1 - \alpha\mathbf{h}_0) \\ &\quad + (\mathbf{h}_2 - \alpha\mathbf{h}_1)^H\mathbf{C}^{-1}(\mathbf{h}_2 - \alpha\mathbf{h}_1) \\ &\quad + \frac{||\mathbf{r}_1 - \mathbf{A}_1\mathbf{h}_1||^2}{\sigma^2} \ . \end{aligned} \tag{14}$$

E_1 can be reduced, dropping constant terms, to

$$E_1 \doteq (\mathbf{h}_1 - \mathbf{g}_1)^H\mathbf{G}_1^{-1}(\mathbf{h}_1 - \mathbf{g}_1) - \mathbf{q}_1^H\mathbf{G}_1\mathbf{q}_1 + \phi_1(\mathbf{h}_2) \tag{15}$$

with

$$\begin{aligned} \mathbf{G}_1 &= \left[\frac{\mathbf{A}_1^H\mathbf{A}_1}{\sigma^2} + (1+|\alpha|^2)\mathbf{C}^{-1}\right]^{-1} \\ \mathbf{q}_1 &= \frac{\mathbf{A}_1^H\mathbf{r}_1}{\sigma^2} + \alpha\mathbf{C}^{-1}\mathbf{h}_0 \\ \mathbf{g}_1 &= \mathbf{G}_1(\mathbf{q}_1 + \alpha^*\mathbf{C}^{-1}\mathbf{h}_2) \ . \end{aligned} \tag{16}$$

After integration over $\mathbf{h}_1$,

$$I_1 \doteq |\mathbf{G}_1| e^{\mathbf{q}_1^H\mathbf{G}_1\mathbf{q}_1} e^{-\phi_1(\mathbf{h}_2)} \ . \tag{17}$$

The term $e^{-\phi_1(\mathbf{h}_2)}$ in (17) is a function of $\mathbf{h}_2$ and will be involved in the integral over $\mathbf{h}_2$. The rest of the terms in (17) are the results after the integration over $\mathbf{h}_1$ and are only decided by the input sequence and will be used to compute the maximum likelihood probability.

The integral over $\mathbf{h}_2$ is

$$I_2 = \int_{\mathbf{h}_2} e^{-E_2} d\mathbf{h}_2 \tag{18}$$

with

$$\begin{aligned} E_2 \quad &= \quad (\mathbf{h}_3 - \alpha\mathbf{h}_2)^H\mathbf{C}^{-1}(\mathbf{h}_3 - \alpha\mathbf{h}_2) + \frac{|r_2 - \mathbf{A}_2\mathbf{h}_2|^2}{\sigma^2} \\ &\quad - |\alpha|^2\mathbf{h}_2^H\mathbf{C}^{-1}\mathbf{G}_1\mathbf{C}^{-1}\mathbf{h}_2 - \alpha\mathbf{q}_1^H\mathbf{G}_1\mathbf{C}^{-1}\mathbf{h}_2 \\ &\quad - \alpha\mathbf{h}_2^H\mathbf{C}^{-1}\mathbf{G}_1\mathbf{q}_1 + \mathbf{h}_2^H\mathbf{C}^{-1}\mathbf{h}_2 \end{aligned} \tag{19}$$

E_2 can be reduced, dropping constant terms, to:

$$E_2 \doteq (\mathbf{h}_2 - \mathbf{g}_2)^H\mathbf{G}_2^{-1}(\mathbf{h}_2 - \mathbf{g}_2) - \mathbf{q}_2^H\mathbf{G}_2\mathbf{q}_2 + \phi_2(\mathbf{h}_3) \tag{20}$$

with

$$\begin{aligned} \mathbf{G}_2 &= \left[\frac{\mathbf{A}_2^H\mathbf{A}_2}{\sigma^2} + \mathbf{C}^{-1} + |\alpha|^2\left(\mathbf{C}^{-1} - \mathbf{C}^{-1}\mathbf{G}_1\mathbf{C}^{-1}\right)\right]^{-1} \\ \mathbf{q}_2 &= \frac{\mathbf{A}_2^H\mathbf{r}_2}{\sigma^2} + \alpha\mathbf{C}^{-1}\mathbf{G}_1\mathbf{q}_1 \\ \mathbf{g}_2 &= \mathbf{G}_2(\mathbf{q}_2 + \alpha^*\mathbf{C}^{-1}\mathbf{h}_3) \end{aligned} \tag{21}$$

After integration over $\mathbf{h}_2$ we have:

$$I_2 \doteq |\mathbf{G}_2| e^{\mathbf{q}_2{}^H\mathbf{G}_2\mathbf{q}_2} e^{-\phi_2(\mathbf{h}_3)} \tag{22}$$

The last term in (22) will be involved in the integral over $\mathbf{h}_3$.

The general form of the results after integral over $\mathbf{h}_i$ ($i = 2, ..., n-1$) will be

$$I_i \doteq |\mathbf{G}_i| e^{\mathbf{q}_i{}^H\mathbf{G}_i\mathbf{q}_i} e^{-\phi_i(\mathbf{h}_{i+1})}, \tag{23}$$

with

$$\begin{aligned} \mathbf{G}_i &= \left[\frac{\mathbf{A}_i^H\mathbf{A}_i}{\sigma^2} + \mathbf{C}^{-1} + |\alpha|^2\left(\mathbf{C}^{-1} - \mathbf{C}^{-1}\mathbf{G}_{i-1}\mathbf{C}^{-1}\right)\right]^{-1} \\ \mathbf{q}_i &= \frac{\mathbf{A}_i^H\mathbf{r}_i}{\sigma^2} + \alpha\mathbf{C}^{-1}\mathbf{G}_{i-1}\mathbf{q}_{i-1}. \end{aligned} \tag{24}$$

For the integral over $\mathbf{h}_n$,

$$I_n \doteq |\mathbf{G}_n| e^{\mathbf{q}_n{}^H\mathbf{G}_n\mathbf{q}_n} \tag{25}$$

with

$$\begin{aligned} \mathbf{G}_n &= \left[\frac{\mathbf{A}_n^H\mathbf{A}_n}{\sigma^2} + \mathbf{C}^{-1} - |\alpha|^2\mathbf{C}^{-1}\mathbf{G}_{n-1}\mathbf{C}^{-1}\right]^{-1} \\ \mathbf{q}_n &= \frac{\mathbf{A}_n^H\mathbf{r}_n}{\sigma^2} + \alpha\mathbf{C}^{-1}\mathbf{G}_{n-1}\mathbf{q}_{n-1} \end{aligned} \tag{26}$$

After the integration over all the channel coefficients, we have the likelihood function

$$f(\mathbf{r^n}|\mathbf{A^n}) \doteq \prod_{i=1}^{n} |\mathbf{G}_i| e^{\mathbf{q}_i{}^H\mathbf{G}_i\mathbf{q}_i}. \tag{27}$$

If we take the negative natural logarithm of the above equation, we will obtain the time-recursive form of the cost function

$$-\log f(\mathbf{r^n}|\mathbf{A^n}) \doteq -\sum_{i=1}^{n}\left[\mathbf{q}_i{}^H\mathbf{G}_i\mathbf{q}_i + \log|\mathbf{G}_i|\right]. \tag{28}$$

From the expressions of $\mathbf{G}_i$ and $\mathbf{q}_i$ we know the transition costs depend on all the previous states, so the standard Viterbi algorithm cannot be applied here. The optimum results can only be obtained by exhaustive search.

4. Reduced Complexity Processing

The multiuser MLSE algorithm derived above requires an exhaustive search of all hypothesized multiuser sequences. The number of hypothesized sequences grows exponentially with time (i.e. as 2^{Kn}). Concerning this problem, one point of view is that, for the unknown channel problem, the Viterbi algorithm can not be used to optimally prune the hypothesized multiuser sequences. This is because received data at time i provides channel estimation information and thus effects the the cost of all hypothesized multiuser sequences up to time i (unless the channel coefficients are statistically independent across time).

Per-Survivor Processing

Per-survivor processing (PSP), [6], has been established as a general, effective, suboptimal approach to the unknown channel problem for pruning the number of sequences considered at any symbol time. Conceptually, for each survivor (unpruned) sequence, the received data is used to estimate the channel (typically data adaptively) under the assumption that that sequence is the correct (desired) signal. In [5], for the single user Gauss-Markov channel MLSE problem, we use a trellis representation on the hypothesized sequences and the list Viterbi algorithm [2] to select the survivors. For each survivor the single-user version of the channel-marginalized MLSE cost derived above is computed. Here we propose the same approach to pruning for the multiuser MLSE estimator, and below we present simulation results using this PSP technique.

Using PSP as described above, the number of multiuser binary sequences kept at Viterbi stage k is $L \cdot 2^{K(M-1)}$, where L is the number of paths kept per trellis state, and $2^{K(M-1)}$ is the number of trellis states. Even with PSP, because of the large number of states, for large K both memory and computational requirements are prohibitive.

Reduced State Sequence Estimation

Reduced State Sequence Estimation (RSSE) (e.g. [8],[9],[10]), is an effective, suboptimal general approach to reducing the number of trellis states to an acceptable number. In the decision-feedback approach to RSSE, tentative symbol decisions are used to reduce the number of unknown symbols represented by the trellis states, thereby reducing the number of trellis states. In the set partitioning approach to RSSE, trellis states are clustered, with each cluster becoming a state in a reduced-state trellis. For the problem considered here, we employ an iterative decision-feedback algorithm to substantially reduce the number of trellis states.

For the multiuser, multipath MLSE problem, at stage i of the full trellis, the $2^{K(M-1)}$ states represent the $K(M-1)$ multiuser bits $\{a_k(i-m);\ k = 1,2,\cdots,K;\ m = 1,2,,\cdots,M-1\}$. For the proposed iterative RSSE approach, K reduced state trellises are set up – one for each user. Each trellis has 2^{M-1} states which for the k^{th} user and the i^{th} stage represent the bits $\{a_k(i-m);\ m = 1,2,\cdots,M-1\}$. At stage i, a sequential update of all K of these users' trellises constitutes an iteration. There are P iterations per each stage of trellises.

Consider the update of the l^{th} user's trellis at stage i and iteration p. For branch costs, soft estimates of the other users' bits, derived from the other trellises, are employed. Denote these soft estimates $\hat{a}_k(i-m);\ m = 1,2,\cdots,M-1;\ k = 1,2,\cdots,K, k \neq l\}$. For the k^{th} user ($k \neq l$), these soft estimates are averages of the bits associated with the L unpruned (list Viterbi) paths from the i^{th} stage of the k^{th} user's reduced trellis, probabilistically weighted by the path likelihoods. For the first iteration and $k > l$, we set $\hat{a}_k(i) = 0$. Because of this, if user power information is available, users should be listed in order of decreasing power.

5. Simulation Results

In this Section we present two sets of simulation results to show the effect that the multiuser PSP and RSSE algorithms presented in this paper have on the performance of MLSE based estimation. For all results, the number of users was $K = 2$, the number of chips per symbol was $N = 4$, the channels each had FIR length $M = 3$, the Gauss-Markov factor was $\alpha = 0.995$, and the Gauss-Markov model noise covariance was $\mathbf{C} = 0.01\ \mathbf{I}$. For each simulation trial a set of the users' chip impulse responses were generated using the Gauss-Markov model. User 1 was 10dB stronger than user 2.

For the first set of results, a full trellis was used (no RSSE), and PSP was compared to the exhaustive search solutions, that provides the optimum MLSE. The number of symbols per simulation was $n = 5$, and 2000 trials per SNR were run. Figure 1 shows results. Although PSP with $L = 1$ path saved per trellis state (i.e. standard Viterbi) performed significantly worse than exact MLSE, performance was close to exact MLSE for $L = 3$. We have found $L = 3$ to be adequate in all simulations we performed.

The second set of results, shown in Figure 2, compare RSSE to full trellis. For both, $L = 3$. The number of symbols per simulation was $n = 5$, and 2000 trials per SNR were run. Figure 2 illustrates that RSSE provides results comparable to full trellis.

6. Summary

In this paper we present new multiuser MLSE algorithms for unknown, fast time-varying channels channels. We model the channels as Gauss-Markov, and marginalize over the channel model parameters to obtain the MLSE cost. For a large number of users and/or channels with deep memory, the optimum, exhaustive search, MLSE algorithm is impractical. We describe a PSP approach, based on list Viterbi, to prune the number sequences kept at each trellis stage. Additional computational savings is realized using a new iterative decision-feedback approach to RSSE.

In the simulation results presented, the proposed PSP/RSSE algorithm performs well compared to optimum and more computationally intensive approximate MLSE solutions. A more extensive performance analysis is in progress.

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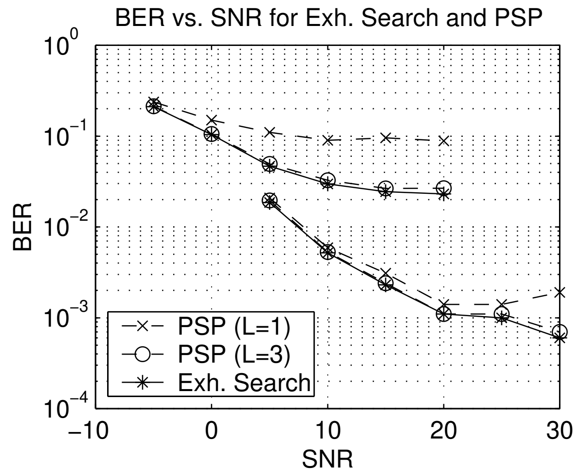

Figure 1: BER vs. SNR for multiuser MLSE with Gauss-Markov channels – exhaustive-search/PSP comparison (user # 1 (lower curves) and user #2).

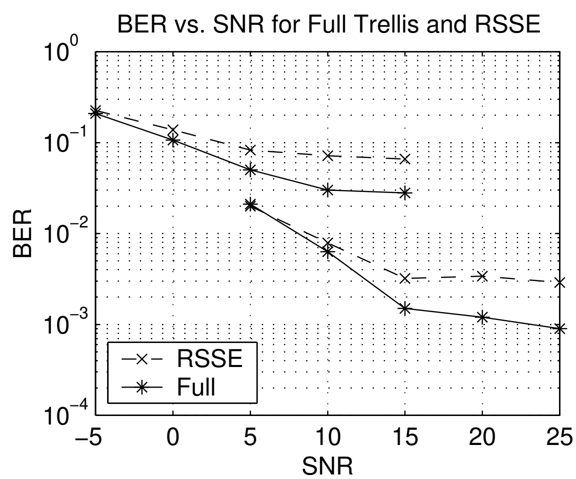

Figure 2: BER vs. SNR for multiuser MLSE with Gauss-Markov channels – Full trellis vs. RSSE (user # 1 (lower curves) and user #2).